Solutions: OCR Core Mathematics C1
January 2007

1

$$\frac{5}{2-\sqrt{3}} = \frac{5}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$
$$= \frac{10+5\sqrt{3}}{4+2\sqrt{3}-2\sqrt{3}-3}$$
$$= \frac{10+5\sqrt{3}}{1}$$
$$= 10+5\sqrt{3}$$

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2 (i) $6^0 = 1$

(ii) $2^{-1} \times 32^{4/5}$:

Write 32 as 2^5.

We then get: $2^{-1} \times (2^5)^{4/5} = 2^{-1} \times 2^4 = 2^3 = 8$

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3 (i) $3(x-5) \le 24$

Expand out brackets: $3x - 15 \le 24$
Add 15: $3x \le 39$
Divide by 3: $x \le 13$

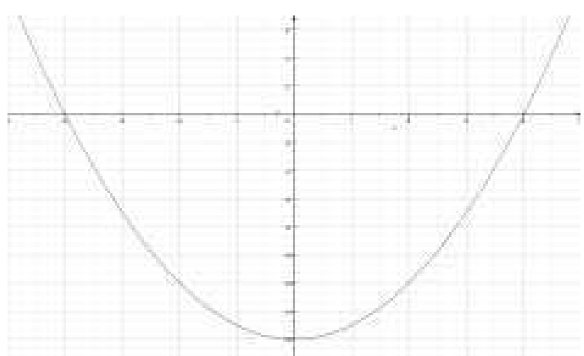
(ii) $5x^2 - 2 > 78$

This is a simple quadratic inequality.

Rearrange to make RHS equal to 0: $5x^2 - 80 > 0$
Divide through by 5: $x^2 - 16 > 0$

Now consider the equivalent quadratic equation: $x^2 - 16 = 0$
This can be factorised (difference of squares): $(x + 4)(x - 4) = 0$
So the solutions are: $x = -4$ or $x = 4$.

To get the solutions for the inequality, consider the graph of $y = x^2 - 16$:



We see $x > 4$ or $x < -4$.

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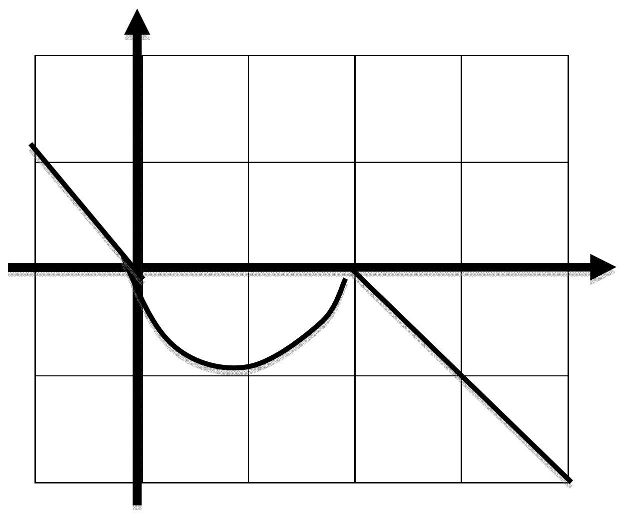
4 The equation $x^{\frac{2}{3}} + 3x^{\frac{1}{3}} - 10 = 0$ is a quadratic equation in disguise.
Use the substitution $y = x^{1/3}$. The equation becomes: $y^2 + 3y - 10 = 0$
This factorises: $(y + 5)(y - 2) = 0$
So the solutions are $y = -5$ or $y = 2$.

To get the solutions for x, we use $x = y^3$: i.e. $x = (-5)^3 = -125$ or $x = 2^3 = 8$.

5 (i) The graph for $y = -f(x)$ is obtained by reflecting in the x-axis.



(ii) The transformation which takes the graph to $y = 3f(x)$ is a stretch scale factor 3 in the y-direction. So the coordinates of Q are (1, 3).

(iii) The transformation which maps the graph to $y = f(x + 2)$ is a translation 2 units to the left.

6 (i) $2x^2 - 24x = 2[x^2 - 12x]$

$$= 2\left[(x-6)^2 - 36\right]$$

$$= 2(x-6)^2 - 72$$

Therefore,

$$2x^2 - 24x + 80 = 2(x-6)^2 - 72 + 80$$

$$= 2(x-6)^2 + 8$$

(ii) The equation of the line of symmetry is $x = 6$. (This is taken from the bracket)

(iii) The equation of the tangent at the minimum point is $y = 8$ (This is because the minimum point has coordinates (6, 8).

7 (i) If $y = 5x + 3$, then $\frac{dy}{dx} = 5$

(ii) If $y = \frac{2}{x^2} = 2x^{-2}$ (write as a negative power), then $\frac{dy}{dx} = -4x^{-3}$.

Rule: Bring down the old power and subtract 1 from the power to get the new power.

(iii) First expand out the brackets: $y = 10x^2 - 9x - 7$

Then differentiate to get: $\frac{dy}{dx} = 20x - 9$

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8 (i) The steps to find the coordinates of the stationary points are:

1) Differentiate the curve to get $\frac{dy}{dx}$;
2) Solve the equation $\frac{dy}{dx}$=0 in order to get the x-coordinates of the stationary points;
3) Find the y-coordinates using the equation of the curve.

Here, $\frac{dy}{dx} = 9 - 6x - 3x^2$.

Therefore we need to solve the equation $9 - 6x - 3x^2 = 0$

To make this equation easier to solve we could first change the signs: $3x^2 + 6x - 9 = 0$

And then we could divide through by 3: $x^2 + 2x - 3 = 0$

This then factorises as: $(x + 3)(x - 1) = 0$

So there are stationary points at x = -3 or x = 1.

We can find the y-coordinates of these points using the formula for the equation of the curve: $y = 27 + 9x - 3x^2 - x^3$

When $x = 1$, $y = 27 + 9 - 3 - 1 = 32$

When $x = -3$, $y = 27 - 27 - 27 + 27 = 0$.

So the stationary points have coordinates (-3, 0) and (1, 32).

(ii) To decide whether the stationary points are maximum or minimum points we follow these steps:

1) Find the second derivative $\frac{d^2y}{dx^2}$
2) Calculate the value of the second derivative at each of the stationary points
3) If $\frac{d^2y}{dx^2} > 0$, then it is a minimum point.

   If $\frac{d^2y}{dx^2} < 0$, then it is a maximum point.

Here, $\frac{d^2y}{dx^2} = -6 - 6x$.

When $x = 1$, $\frac{d^2y}{dx^2} = -6 - 6(1) = -12 < 0$ so a maximum point.

When $x = -3$, $\frac{d^2y}{dx^2} = -6 - 6(-3) = 12 > 0$ so a minimum point.

(iii) To decide where the curve is increasing, it is helpful to use the coordinates of the stationary points to help us to sketch the curve:

We see that it is increasing for $-3 < x < 1$.

9 (i) Parallel lines have the same gradient.
The line y = 4x – 5 has gradient 4.
So a parallel line also has gradient 4.

The equation of a line with gradient m passing through the point (a, b) is

$$y - y_1 = m(x - x_1).$$

As our line must pass through the point (2, 7) its equation must be:

$$y - 7 = 4(x - 2)$$

i.e. y – 7 = 4x – 8
i.e. y = 4x – 1.

(ii) You can calculate the distance between two points EITHER by sketching a diagram and using Pythagoras' theorem OR by using the following result:

The distance between the points (x_1, y_1), (x_2, y_2) is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

If we use this formula here, the distance between A(2, 7) and B(-1, -2) is:

$$\sqrt{(-1-2)^2 + (-2-7)^2} = \sqrt{(-3)^2 + (-9)^2} = \sqrt{90}$$

This can be simplified to make $3\sqrt{10}$

(iii) The gradient of AB is $\frac{-2-7}{-1-2} = \frac{-9}{-3} = 3$
The midpoint of AB is (0.5, 2.5)

The gradient of a perpendicular line is $\frac{-1}{3}$ (i.e. the negative reciprocal).
So the equation of a perpendicular line is: $y - \frac{5}{2} = -\frac{1}{3}(x - \frac{1}{2})$.

Multiply by 6: $6y - 15 = -2(x - \frac{1}{2})$
i.e. $6y - 15 = -2x + 1$
The equation therefore is $2x + 6y - 16 = 0$
Or $x + 3y - 8 = 0$.

10 (i) The circle has equation $x^2 + y^2 + 2x - 4y - 8 = 0$
To find the coordinates of the centre and the value of the radius, we need to rewrite the equation in the form $(x-a)^2 + (y-b)^2 = r^2$

We do this using completing the square:

$$x^2 + 2x = (x+1)^2 - 1$$
$$y^2 - 4y = (y-2)^2 - 4$$

The equation of the circle therefore is: $(x+1)^2 - 1 + (y-2)^2 - 4 - 8 = 0$
Or $(x+1)^2 + (y-2)^2 = 13$

The circle has centre (-1, 2) and radius √13.

(ii) Substitute the coordinates (-3, k) into either equation of the circle.
The algebra is easier if we substitute into $(x+1)^2 + (y-2)^2 = 13$.
We get: $(-3+1)^2 + (k-2)^2 = 13$
i.e. $4 + (k-2)^2 = 13$
i.e. $(k-2)^2 = 9$

Square rooting both sides: $k - 2 = \pm 3$
So the solutions are $k = 5$ or $k = -1$.

As $k < 0$, the only solution is $k = -1$.

(iii) The equation of the line is $x + y = 6$ OR $y = 6 - x$.

If we substitute this into the equation of the circle $x^2 + y^2 + 2x - 4y - 8 = 0$, we get:

$$x^2 + (6-x)^2 + 2x - 4(6-x) - 8 = 0$$

Expanding out the brackets gives:

$$x^2 + 36 - 12x + x^2 + 2x - 24 + 4x - 8 = 0$$

Simplifying:

$$2x^2 - 6x + 4 = 0$$

Divide by 2:

$$x^2 - 3x + 2 = 0$$

This factorises as (x – 1)(x – 2) = 0 so the solutions are $x = 1$, $x = 2$.

When $x = 1$, $y = 5$
When $x = 2$, $y = 4$

Therefore the coordinates of the points of intersection are (1, 5), (2, 4).

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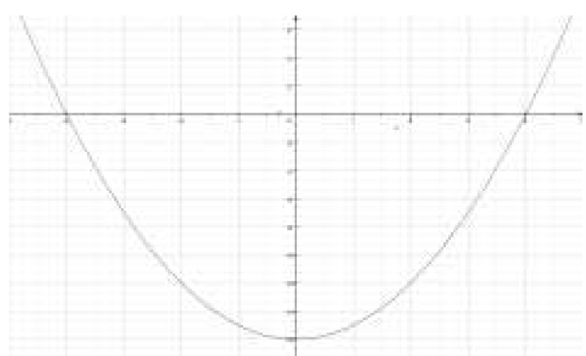

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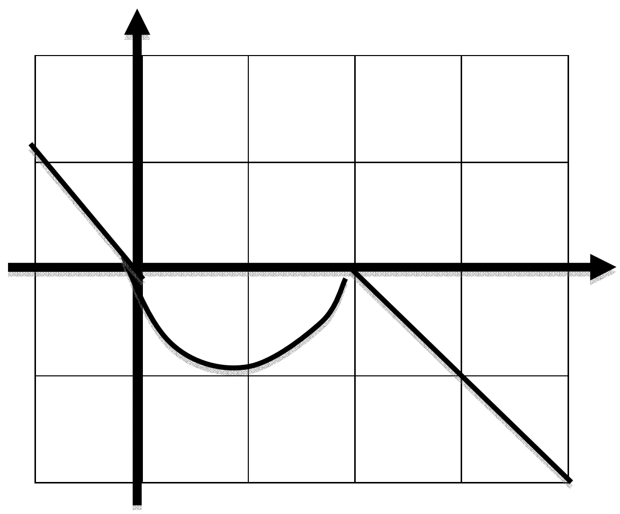

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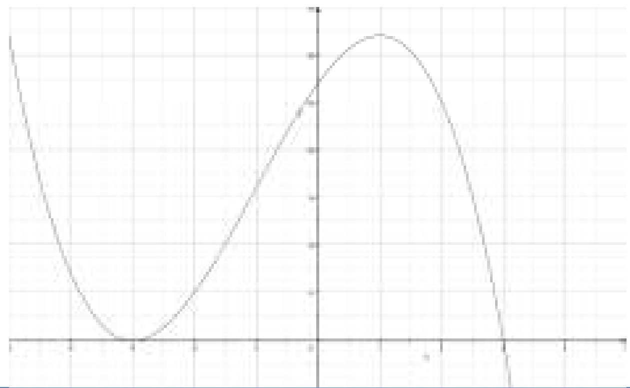

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