Solutions: OCR Core Mathematics 2
January 2007

1 The relevant formulae for this question are:

The nth term in an AP is given by the formula $a+(n-1)d$

The sum of the first n terms of an AP is $S_n = \frac{1}{2}n[2a+(n-1)d]$

These formulae are both given in the formula book.

Given information:	first term is 15	i.e. a = 15
	20^{th} term is 72	i.e. 72 = a + 19d

This means:
72 = 15 + 19d
57 = 19d
d = 3

So the sum of the first 100 terms is

$$S_{100} = \frac{1}{2}\times 100 \times [2\times 15 + (99)\times 3] = 50\times 327 = 16350$$

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2 (i) The key relationship between degrees and radians is $180^\circ = \pi$ radians

So $1^\circ = \frac{\pi}{180}$ radians

i.e. $46^\circ = \frac{46\pi}{180}$ or 0.803 radians

(ii) The formula for arc length is $s = r\theta$ provided that the angle θ is in radians.
So here s = 8 × 0.803 = 6.42 cm

(iii) The formula for the area of a sector is $A = \frac{1}{2}r^2\theta$ provided that the angle θ is in radians.

So, area = $\frac{1}{2}\times 8^2 \times 0.803 = 25.7$ cm^2.

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3 (i) $\int (4x-5)dx = \frac{4}{2}x^2 - 5x + c = 2x^2 - 5x + c$

(Remember the rule for integrating: add one to the power and divide by the new power. Remember the constant of integration)

(ii) To find the equation of the curve, we have to undo the differentiation by integrating.
From part (i) we get:

$y = 2x^2 - 5x + c$

But we know that the curve passes through the point (3, 7). We can use this information in order to find c:

$7 = 2\times 3^2 - 5\times 3 + c$

i.e. 7 = 3 + c

so c = 4.

Therefore the curve is $y = 2x^2 - 5x + 4$

4 (i) The formula for the area of a triangle is $A = \frac{1}{2}ab\sin C$ or $A = \frac{1}{2}ac\sin B$.

This formula for the area relies upon two sides and the included angle. Note that you have to learn the structure of the formula.

So: area $= \frac{1}{2} \times 5\sqrt{2} \times 8 \times \sin 60 = \frac{1}{2} \times 5\sqrt{2} \times 8 \times \frac{\sqrt{3}}{2}$

$= \frac{40}{4}\sqrt{6} = 10\sqrt{6}\ \text{cm}^2$.

(ii) The formula for the cosine rule is: $a^2 = b^2 + c^2 - 2bc\cos A$ (this is in the formula book).
However here we wish to find side b and we know angle B.
Therefore the version of the cosine rule that is used here is $b^2 = a^2 + c^2 - 2ac\cos B$

So:

$$b^2 = 8^2 + (5\sqrt{2})^2 - 2 \times 8 \times 5\sqrt{2}\cos 60$$
$$= 57.4315$$

i.e. b = 7.58 cm

Note: type the calculation for the cosine rule into your calculator in one go – many students are unable to substitute into the cosine rule correctly.

5 a) (i) $\log_3(4x+7) - \log_3 x = \log_3\left(\frac{4x+7}{x}\right)$

Note: this uses the rule $\log_a m - \log_a n = \log_a\left(\frac{m}{n}\right)$ - LEARN THIS!

(ii) The equation can be rewritten as $\log_3\left(\frac{4x+7}{x}\right) = 2$

We can undo the logarithm to the base 3 by taking powers of 3 of both sides:
We get:

$$\frac{4x+7}{x} = 3^2$$

Multiply through by x:

$$4x + 7 = 9x$$

i.e. 7 = 5x
i.e. x = 1.4

b) We need to draw up a table of values. The width of each interval is 3 so our table has x coordinates which increase by 3 each time.

x	3	6	9
y	$\log_{10} 3$	$\log_{10} 6$	$\log_{10} 9$

The formula for the trapezium rule (given in the formula book) is

$$\int_a^b y\,dx \approx \frac{h}{2}[y_0 + y_n + 2(y_1 + \ldots + y_{n-1})]$$

Each y coordinates gets substituted into this formula in one position only.

So:

$$\int_3^9 \log_{10} x dx \approx \frac{3}{2}\left[\log_{10} 3 + \log_{10} 9 + 2\times\log_{10} 6\right] = 1.5\times 2.9877$$

$$= 4.48$$

6 (i) To find the binomial expansion, we need both the binomial coefficients and the powers of 1 and 4x:

Binomial coefficient	Powers	Term
${}^7C_0 = 1$	$1^7 = 1$	1
${}^7C_1 = 7$	$1^6(4x) = 4x$	28x
${}^7C_2 = 21$	$1^5(4x)^2 = 16x^2$	$336x^2$
${}^7C_3 = 35$	$1^4(4x)^3 = 64x^3$	$2240x^3$

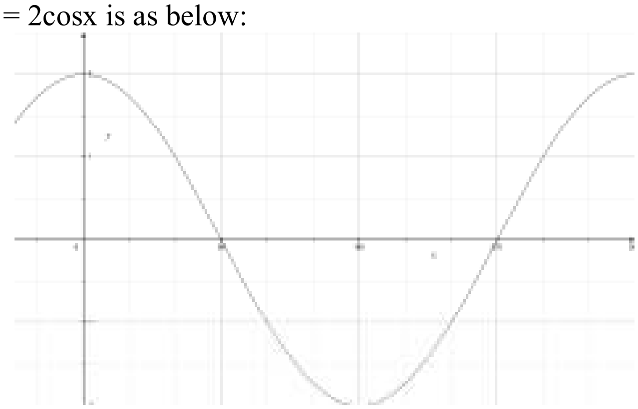
Therefore $(1 + 4x)^7 = 1 + 28x + 336x^2 + 2240x^3 + \ldots$

(ii) Consider the expansion of $(3+ax)(1+4x)^7$.
Using the answer to part (i), this is the same as $(3+ax)(1+28x+336x^2+2240x^3+...)$.

The coefficient of x^2 in this expansion is $336\times 3 + 28a$.

Therefore we have the equation: $1001 = 1008 + 28a$
i.e. $-7 = 28a$
i.e. $a = -1/4$.

7 (i) a) The graph of y = 2cosx is as below:



b) 2cosx = 0.8
Divide by 2: cosx = 0.4

So: x = 66.4° (from calculator) or x = 360 – 66.4 = 293.6°

(ii) The key result here is: $\tan x = \frac{\sin x}{\cos x}$ (LEARN THIS)
So to solve $2\cos x = \sin x$, we first divide both sides by cosx:
2 = tanx.

Therefore, x = 63.4° or x = 180 + 63.4 = 243.4°.
The second solution is outside the required range. We can find an equivalent angle by

subtracting 360 degrees.
So the solutions are x = 63.4° or -116.6°

8 (i) The remainder theorem states that the remainder when a polyonomial f(x) is divided by (x + a) is f(-a).

So the remainder when we divide by (x + 2) is given by f(-2);

$$f(-2) = (-2)^3 - 9(-2)^2 + 7(-2) + 33 = -25$$

(ii) According to the factor theorem, to show that (x – 3) is a factor of f(x), we need to show f(3) = 0.

$f(3) = (3)^3 - 9(3)^2 + 7(3) + 33 = 27 - 81 + 21 + 33 = 0$ as required.

9 (i) First trip: 1.5 tonnes
2nd trip: 1.5 × 1.02 = 1.53 tonnes
3rd trip: 1.53 × 1.02 = 1.5606 tonnes
4th trip: 1.5606 × 1.02 = 1.5918 tonnes
5th trip: 1.5918 × 1.02 = 1.624 tonnes.

Note: a more efficient way to calculate the 5th term would be to utilise the formula for the nth term of a GP:

$$5^{\text{th}} \text{ term} = ar^4 = 1.5 \times 1.02^4 = 1.624$$

(ii) If there are 39 tonnes of coal available, then the total amount of coal must not exceed 39. Therefore we need to find N such that the sum of the first N terms must be less than (or equal to) 39.

The formula for the sum of N terms of a GP is $S_N = \dfrac{a(1-r^N)}{1-r}$.

Substituting in a = 1.5, r = 1.02 gives:

$$S_N = \frac{1.5(1-1.02^N)}{1-1.02} \le 39$$

i.e $\dfrac{1.5(1-1.02^N)}{-0.02} \le 39$

So: $1.5(1-1.02^N) \ge 39 \times -0.02$

i.e $(1-1.02^N) \ge -0.52$

So: $-1.02^N \ge -1.52$

i.e $1.02^N \le 1.52$

(iii) Take logs of both sides: $\log(1.02^N) \le \log(1.52)$

So: $N\log(1.02) \le \log(1.52)$

i.e. $N \le \log(1.52) \div \log(1.02)$

So: $N \le 21.14$

The greatest number of trips is therefore 21.

10 (i) The curve is $y = 1 - 3x^{-1/2}$

Substitute in x = 9: $y = 1 - 3\times 9^{-1/2} = 0$.

Therefore the curve intersects the x-axis at (9, 0).

(ii) The shaded area is given by the integral:

$$\int_9^a (1 - 3x^{-1/2})dx = \left[x - \frac{3}{1/2}x^{1/2}\right]_9^a = \left[x - 6x^{1/2}\right]_9^a$$

(remembering to add 1 to the power and to divide by the new power).

Substituting in the two limits, we get that the shaded area is:

$$\left(a - 6a^{1/2}\right) - \left(9 - 6\times 9^{1/2}\right) = a - 6\sqrt{a} + 9.$$

We are told the shaded area is 4. So we can form an equation:

$$a - 6\sqrt{a} + 9 = 4$$

or $$a - 6\sqrt{a} + 5 = 0$$

This is a quadratic equation involving $\sqrt{a}$. Let m = $\sqrt{a}$.

The equation then becomes $m^2 - 6m + 5 = 0$

$(m - 1)(m - 5) = 0$

So $m = 1$ or $m = 5$.

Therefore, as $a = m^2$, we must have $a = 25$ (since $a > 9$).