Solutions: OCR Core Mathematics C2
June 2006

1 $(3x-2)^4$:

To find the binomial, we need the binomial coefficients and the powers of 3x and -2:

Binomial coefficient	Powers	Term
$^4C_0 = 1$	$(3x)^4 = 81x^4$	$81x^4$
$^4C_1 = 4$	$(3x)^3(-2) = -54x^3$	$-216x^3$
$^4C_2 = 6$	$(3x)^2(-2)^2 = 36x^3$	$216x^2$
$^4C_3 = 4$	$(3x)^1(-2)^3 = -24x$	$-96x$
$^4C_4 = 1$	$(-2)^4 = 16$	16

So the expansion is $81x^4 - 216x^3 + 216x^2 - 96x + 16$

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2 (i)

$$u_1 = 2$$
$$u_{n+1} = 1 - u_n$$

This sequence has been defined inductively. The next few terms in this sequence are:

$$u_2 = 1 - u_1 = 1 - 2 = -1$$
$$u_3 = 1 - u_2 = 1 - (-1) = 2$$
$$u_4 = 1 - u_3 = 1 - 2 = -1$$

(ii) The sequence alternates in value between 2 and -1.
We are now adding up the first 100 terms in this sequence.
50 of the terms will be 2 – these add up to make 100
50 of the terms will be -1 – these add up to make -50.
So the overall total will be 100 – 50 = 50.

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3 The gradient function is $\frac{dy}{dx} = 2x^{-1/2}$.

To find the equation of the curve, we will need to integrate (to undo the differentiation).

So, $y = \int 2x^{-1/2}dx = \frac{2}{1/2}x^{1/2} + c$ (remember add one to the power and divide by the new power)

i.e. $y = 4\sqrt{x} + c$

The curve passes through the point (4, 5). This piece of information is there to help us find the constant of integration. Substitute x = 4, y = 5 into the equation of the curve.

$$5 = 4\sqrt{4} + c$$
$$5 = 8 + c$$
$$c = -3$$

Therefore the curve has equation, $y = 4\sqrt{x} - 3$

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4 (i) To find where the line and the curve intersect, we substitute $y = x + 2$ into the equation of the curve:

$$x + 2 = 4 - x^2$$

Rearrange to make the right hand side equal to 0:

$$x^2 + x - 2 = 0$$

Factorise:

$$(x + 2)(x - 1) = 0$$

i.e. $x = 1$ or $x = -2$.

(ii) To find the area trapped between two curves, we can use the result:

trapped area $= \int$ (top curve) - (bottom curve) dx

So here need to work out:

$$\int_{x=-2}^{1} (4 - x^2) - (x + 2) dx$$

$$= \int_{-2}^{1} (2 - x^2 - x) dx$$

$$= \left[2x - \frac{1}{3}x^3 - \frac{1}{2}x^2 \right]_{-2}^{1}$$

We now substitute in the two limits and SUBTRACT. We get:-

$$\left(2 - \frac{1}{3} - \frac{1}{2}\right) - \left(-4 - \frac{1}{3}(-8) - \frac{1}{2}(-2)^2\right)$$

$$= \left(1\frac{1}{6}\right) - \left(-3\frac{1}{3}\right)$$

$$= 4\frac{1}{2}$$

5 (i) The first equation uses the following key result (which you must learn):

$$\sin^2\theta + \cos^2\theta = 1$$

We can use this result here to rewrite $\sin^2 x$ as $1 - \cos^2 x$:

$$2(1 - \cos^2 x) = 1 + \cos x$$

$$2 - 2\cos^2 x = 1 + \cos x$$

$$2\cos^2 x + \cos x - 1 = 0$$

This is a quadratic equation in cos$x$. Let y = cos$x$. The equation then becomes:

$$2y^2 + y - 1 = 0$$

i.e. $(2y - 1)(y + 1) = 0$

i.e. $y = 0.5$ or $y = -1$.

So

$\cos x = 0.5 \quad \rightarrow \quad x = 60^\circ$ (only solution in given interval)

or $\cos x = -1 \quad \rightarrow \quad x = 180^\circ$

(ii) The second equation requires you to know the second key relationship in trigonometry:

$$\frac{\sin\theta}{\cos\theta} = \tan\theta \quad \text{(Learn this!)}$$

The equation to be solved is: $\sin 2x = -\cos 2x$

Divide both sides by $\cos 2x$:

$$\frac{\sin 2x}{\cos 2x} = -1$$

i.e $\quad \tan 2x = -1$

We can solve this equation by first finding the solutions for $2x$ and then dividing these by 2 to get the solutions for x:

	Solutions for $2x$	Solutions for x
	$(\tan^{-1} 1 = 45^\circ)$	
	$2x = 180 - 45 = 135^\circ$	$x = 135/2 = 67.5^\circ$
or	$2x = 360 - 45 = 315^\circ$	$x = 315/2 = 157.5^\circ$

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6 (i) John's payments form an arithmetic progression with $a = 100$ and $d = 5$.

These are the two key formulae relating to arithmetic progressions:

nth term $= a + (n-1)d$

Sum of n terms $= \frac{n}{2}[2a + (n-1)d]$

a) Payment in 240th month $= a + 239d = 100 + 239 \times 5 = 1295$
Therefore his final payment is £1295.

b) Total of all payments = sum of 240 terms = $\frac{240}{2}[2 \times 100 + 239 \times 5] = 167400$
So altogether he will have paid in £167400.

(ii) Rachel's payments form a geometric progression with $a = 100$, but the common ratio is unknown.

The two key formulae for geometric progressions are:

nth term $= ar^{n-1}$

Sum of n terms $= \frac{a(1-r^n)}{1-r}$

We are told that the 240th term is £1500. This piece of information will enable use to find the common ratio, r:

$$240^{\text{th}} \text{ term} = ar^{239}$$
$$1500 = 100r^{239}$$
$$15 = r^{239}$$

Therefore $\quad r = \sqrt[239]{15} = 1.0113952$

The sum of 240 terms $= \frac{100(1 - 1.0113952^{240})}{1 - 1.0113952} = £124359$
So Rachel will have paid in £124000 (to 3 SF)

7(i) We can use the cosine rule to find the length of AC:

$AC^2 = 8^2 + 11^2 - 2 \times 8 \times 11 \times \cos 0.8$ (set calculator to radians!)

$AC^2 = 185 - 122.62 = 62.38$
$AC = 7.90$ cm

(ii) The formula for the area of a sector is $\frac{1}{2}r^2\theta$

Area of sector ACD = $\frac{1}{2} \times 7.90^2 \times 1.7$=53.0485

The formula for the area of a triangle is $\frac{1}{2}ab\sin C$

Area of triangle ACD = $\frac{1}{2}7.9 \times 7.9\sin 1.7$=30.9449

So shaded area = 53.0485 – 30.9449 = 22.1 cm^2.

(iii) The formula for the arc length is $r\theta$.
So the arc length CD = $7.9 \times 1.7 = 13.43$

Using the cosine rule the length of the straight side CD is:

$$CD^2 = 7.9^2 + 7.9^2 - 2 \times 7.9 \times 7.9 \times \cos 1.7$$

$$CD^2 = 140.902$$

$$CD = 11.9cm$$

So the perimeter is 13.43 + 11.9 = 25.3 cm

8 (i) The remainder theorem says that the remainder when a polynomial f(x) is divided by ($x – a$) is f(a).

The factor theorem says that if ($x – a$) is a factor of a polynomial f(x), then f(a) = 0.

Here, $f(x) = 2x^3 + ax^2 + bx - 10$

<u>Information 1</u>: The remainder when f(x) is divided by (x – 2) is 12.
This means that f(2) = 12.
So, $2 \times 2^3 + a \times 2^2 + 2b - 10 = 12$
i.e. 16 + 4a + 2b – 10 = 12
i.e. 4a + 2b = 6
i.e. 2a + b = 3. (1)

Information 2: (x + 1) is a factor of f(x).
This means that f(-1) = 0.
So, $2 \times (-1)^3 + a \times (-1)^2 - b - 10 = 0$
i.e. -2 + a – b – 10 = 0
i.e. a – b = 12 (2)

We therefore have two simultaneous equations. If we add these together, we get:

3a = 15 i.e. a = 5

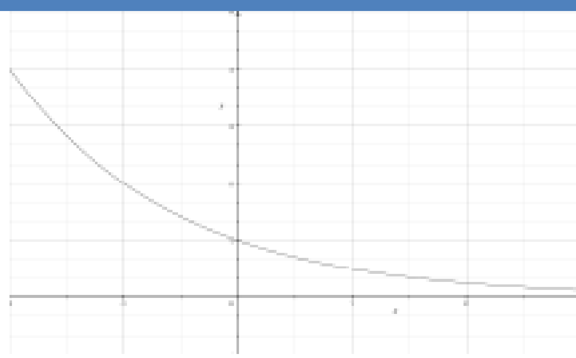
So b = -7.

(ii) We get the quotient and remainder by doing a long division:

$$
\begin{array}{r}
2x^2 + x - 9 \\
x+2\,\overline{\big)\,2x^3 + 5x^2 - 7x - 10} \\
\underline{2x^3 + 4x^2} \qquad\qquad \\
x^2 - 7x \qquad \\
\underline{x^2 + 2x} \qquad \\
-9x - 10 \\
\underline{-9x - 18} \\
8
\end{array}
$$

So the remainder is 8 and the quotient is $2x^2 + x - 9$.

9 (i)



The curve crosses the y-axis at (0, 1).

(ii) We draw up a table of values. We use x-values from $x = 0$ to $x = 2$ (in steps of 0.5):

x	0	0.5	1	1.5	2
y	1	0.7071	0.5	0.3536	0.25

We substitute the y-coordinates into the formula for the trapezium rule:

$$\int y\,dx \approx \frac{h}{2}\left[y_0 + y_n + 2(y_1 + \ldots + y_{n-1})\right]$$

$$= \frac{0.5}{2}\left[1 + 0.25 + 2(0.7071 + 0.5 + 0.3536)\right]$$

$$= 1.09 \quad (3\text{ SF})$$

(iii) We can form an equation:

$$\left(\frac{1}{2}\right)^x = \frac{1}{6}$$

i.e. $$\frac{1}{2^x} = \frac{1}{6}$$

Therefore, $2^x = 6$

Take logs of both sides:

$$\log_{10} 2^x = \log_{10} 6$$

$$x \log_{10} 2 = \log_{10} 6$$

$$x = \frac{\log_{10} 6}{\log_{10} 2}$$

So $$x = \frac{\log_{10}(2 \times 3)}{\log_{10} 2} = \frac{\log_{10} 2 + \log_{10} 3}{\log_{10} 2} = 1 + \frac{\log_{10} 3}{\log_{10} 2}$$ (as required)