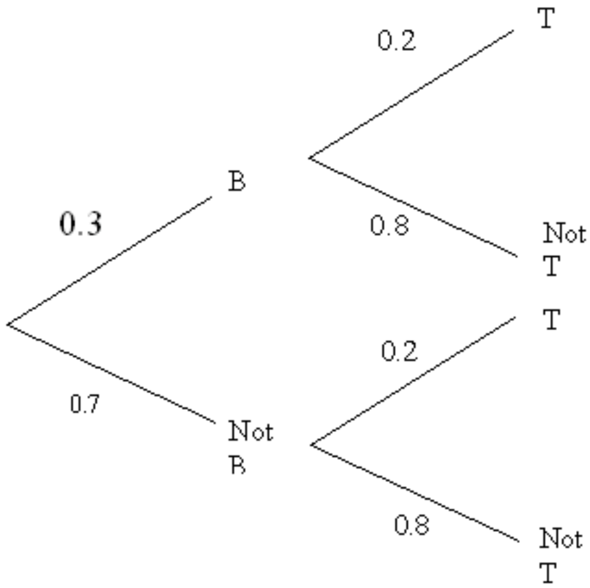
Schoolworkout Maths

Recap: Independent Events

Example 1: I travel by bus and train on my journey to work. Given that P(bus late) is 0.3 and independently P(train late) is 0.2, calculate the probability that:

a) both the bus and the train are late;
b) either the bus or the train (or both) are late.

We can answer this question by drawing a tree diagram. Let B = bus late, T = train late.



a) P(B and T) = $0.3 \times 0.2 = 0.06$

b) P(B OR T) = 1 – P(neither are late) = $1 - 0.7 \times 0.8 = 0.44$.

Example 2: Caroline plays a game with a fair coin. She keeps throwing the coin until a head is obtained, at which point she stops. Calculate the probability that:

a) the game finishes on the 3rd throw;
b) the game finishes in less than 5 attempts;
c) the game continues for more than 10 throws.

a) P(game finishes on 3rd go) = P(Tail THEN Tail THEN Head) = $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$

b) P(game finishes in less than 5 attempts) = P(finishes on 1st or 2nd or 3rd or 4th go)

But, P(game finishes on 1st go) = P(Head) = $\frac{1}{2}$

P(game finishes on 2nd go) = P(T AND H) = $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

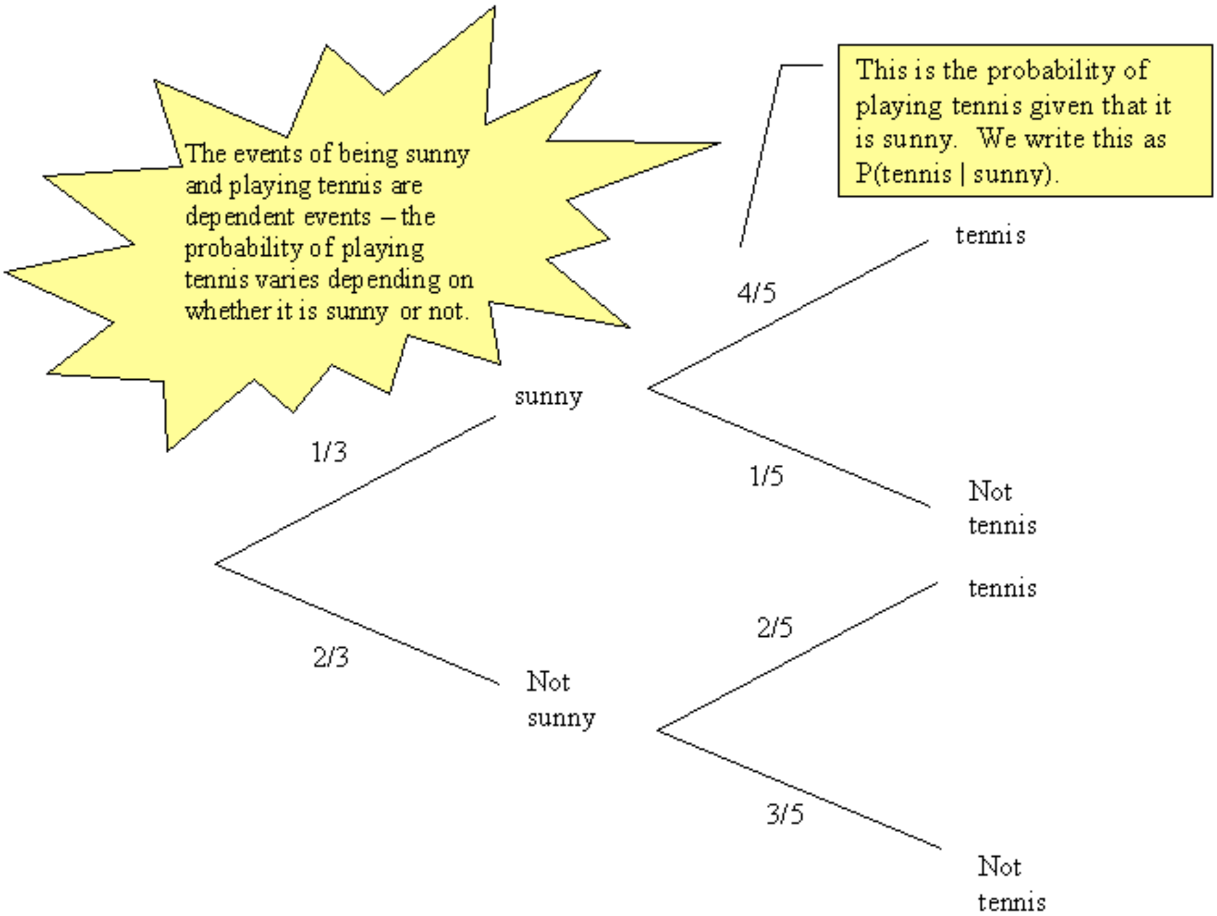
P(game finishes on 4th go) = P(T, T, T, H) = $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{16}$

So P(game finishes in less than 5 goes) = $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}=\frac{15}{16}$

c) P(game continues for more than 10 throws) = P(first 10 throws are tails) = $\left(\frac{1}{2}\right)^{10}=\frac{1}{1024}$

Dependent Events and Conditional Probability

Introductory example: The probability that it will be sunny tomorrow is $\frac{1}{3}$. If it is sunny, the probability that Susan plays tennis is $\frac{4}{5}$. If it is not sunny, the probability that Susan plays tennis is $\frac{2}{5}$. Find the probability that Susan plays tennis.



P(tennis) = P(sunny AND tennis) + P(not sunny AND tennis)

$$= \frac{1}{3}\times\frac{4}{5}+\frac{2}{3}\times\frac{2}{5}=\frac{8}{15}$$

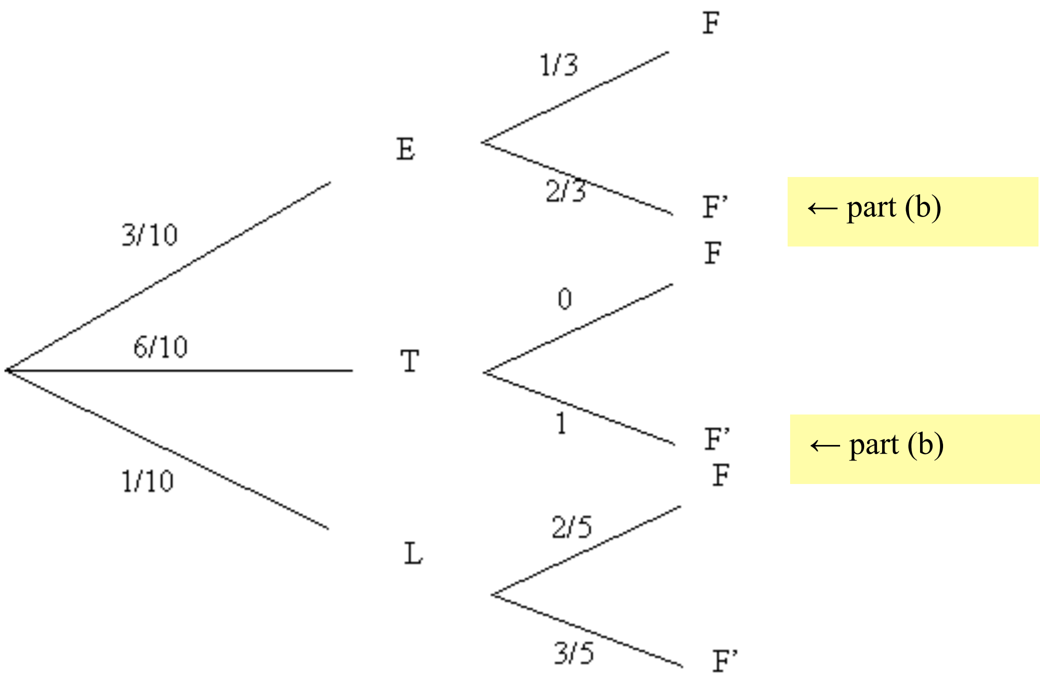
Example 2: On average I get up early 3 days out of 10, and get up late one day in 10. I forget something on 2 out of every 5 days on which I am late and on one third of the days on which I am early. I do not forget anything on the days when I get up on time.

(a) Find the probability that I forget something.

(b) Confirm that for 80% of the time I am neither forgetful nor late.

Let E = get up early
T = get up on time
L = get up late
F = forget something
F' = don't forget anything

The tree diagram then looks as follows:



a) P(forget something) = P(E and F) + P(T and F) + P(L and F)

$$= \left(\frac{3}{10}\times\frac{1}{3}\right)+\left(\frac{6}{10}\times 0\right)+\left(\frac{1}{10}\times\frac{2}{5}\right)=\frac{7}{50}$$

b) The two situations where I am neither forgetful nor late have been indicated on the tree diagram:

So P(neither forgetful of late) $= \frac{3}{10}\times\frac{2}{3}+\frac{6}{10}\times 1=\frac{4}{5}$

Therefore, on 80% of occasions I will neither forget anything nor be late.