S1 Revision Notes: Probability

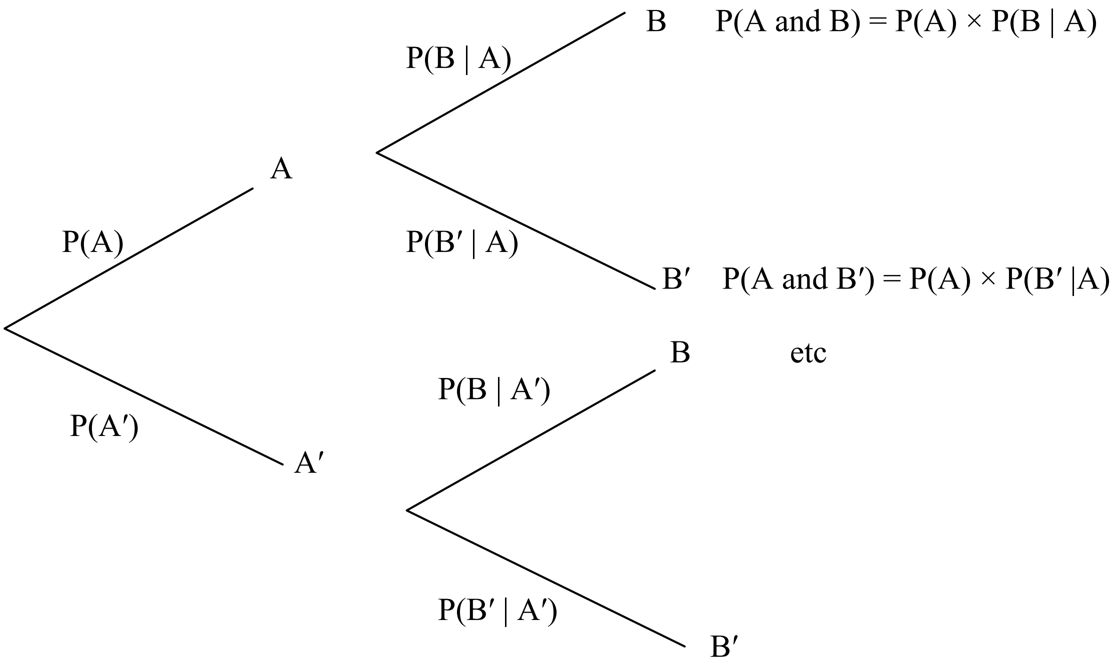
Recap:

- **Conditional probability**

$$\textbf{P(A and B)} = \textbf{P(A)} \times \textbf{P(B | A)}$$

The probability of B happening given that A has already happened.

This result relates to a tree diagram:



Special case: If A and B are independent events (so that the occurrence of A does not affect the occurrence of B) then P(B | A) = P(B). So in this situation, we have

$$\textbf{P(A and B)} = \textbf{P(A)} \times \textbf{P(B)}$$

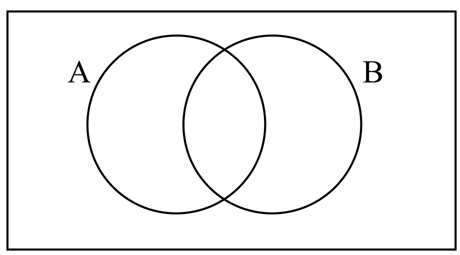
Note: The formula at the top of the page rearranges to give

$$\text{P}(\text{B}\,|\,\text{A}) = \frac{\text{P(A and B)}}{\text{P(A)}}$$

- **Addition Rule**

$$\textbf{P(A or B)} = \textbf{P(A)} + \textbf{P(B)} - \textbf{P(A and B)}$$

This result relates to a Venn Diagram:



Special case: If A and B are mutually exclusive (i.e. cannot occur at the same time), then P(A and B) = 0. In this situation, we have

$$\mathbf{P(A \text{ or } B) = P(A) + P(B).}$$

All probability questions can be answered using the formulae and diagrams shown on the previous page. Probability questions however usually need some thought.

- **Example:**

Jackie attends college on Monday, Tuesday, Thursday and Friday each week. The probability of her being late on a particular day is shown in the table below and is independent of whether she was late on any previous day.

Day	Monday	Tuesday	Thursday	Friday
Probability late	0.4	0.2	0.2	0.3

What is the probability that during a particular week Jackie is:

a) late every day;
b) not late at all;
c) late on exactly one day?

The probability that Jackie's friend Curtley is late is 0.1 if Jackie is not late and is 0.6 if Jackie is late.

What is the probability that on a particular Monday:

d) both Jackie and Curtley are late;
e) Curtley is late but Jackie is not;
f) Curtley is late;
g) Jackie is not late given that Curtley is late?

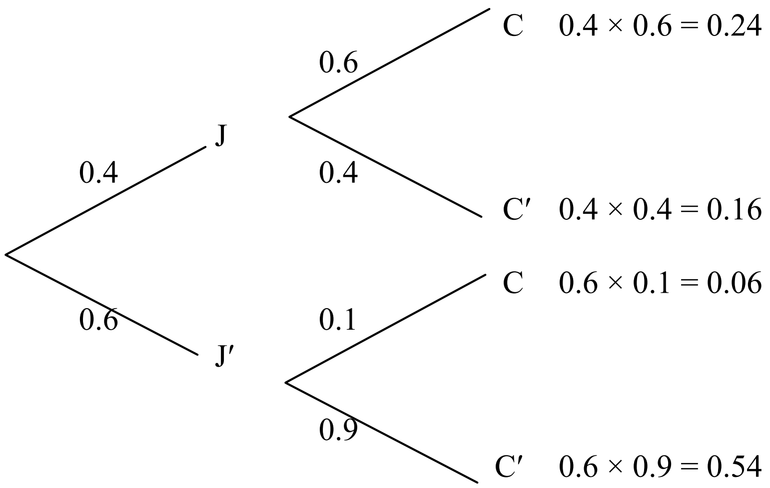
- **Solution**

a) P(late on Monday, Tuesday, Thursday and Friday) = $0.4 \times 0.2 \times 0.2 \times 0.3 = 0.0048$.

b) P(not late on every day) = $0.6 \times 0.8 \times 0.8 \times 0.7 = 0.2688$.

c) P(late exactly once) = P(Late only on Monday) + P(late only on Tuesday) + P(late only on Thursday) + P(late only on Friday)

$$= (0.4 \times 0.8 \times 0.8 \times 0.7) + (0.6 \times 0.2 \times 0.8 \times 0.7) + (0.6 \times 0.8 \times 0.2 \times 0.7) + (0.6 \times 0.8 \times 0.8 \times 0.3)$$
$$= 0.1792 + 0.0672 + 0.0672 + 0.1152$$
$$= 0.4288$$



J = Jackie late
C = Curtley late

Using the tree diagram:-

d) P(both Jackie and Curtley late) = 0.24

e) P(Curtley late but Jackie not) = 0.06

f) P(Curtley late) = 0.24 + 0.06 = 0.3

g) P(Jackie not late | Curtley late) $= \dfrac{\text{P(Jackie not late and Curtley late)}}{\text{P(Curtley late)}}$

$$= \frac{0.06}{0.3} = 0.2$$

- **Example:**

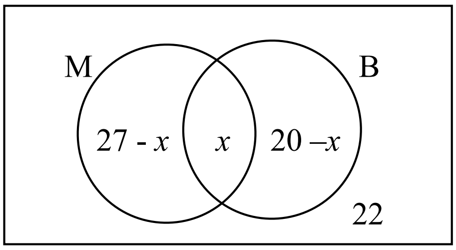
There are 60 students in the sixth form of a certain school. Mathematics is studied by 27 of them, biology by 20 and 22 study neither mathematics nor biology.

a) Find the probability that a randomly selected student studies both mathematics and biology.
b) Find the probability that a randomly selected mathematics student does not study biology.

A student is selected at random.

c) Determine whether the event "studying mathematics" is statistically independent of the event "studying biology".

- **Solution**



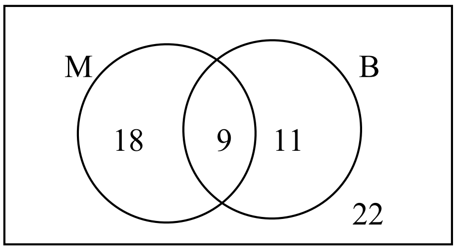
Let the number of students studying both maths and biology be x.

a) Total number of students = 60

$$\text{So } 27 - x + x + 20 - x + 22 = 60$$
$$69 - x = 60$$
$$x = 9$$

So P(student studies both) = $\dfrac{9}{60} = \dfrac{3}{20}$

b) P(maths student doesn't study biolgy) = $\dfrac{18}{27} = \dfrac{2}{3}$



c) If two events are independent, P(maths and biology) = P(maths) × P(biology).

From the Venn Diagram, P(maths and biology) = $\dfrac{3}{20}$

P(maths) = $\dfrac{27}{60} = \dfrac{9}{20}$

P(biology) = $\dfrac{20}{60} = \dfrac{1}{3}$

As $\dfrac{9}{20} \times \dfrac{1}{3} = \dfrac{3}{20}$, the events are independent.

Example
An urn contains 3 red, 4 white and 5 blue discs. Three discs are selected at random from the urn. Find the probability that all three discs are of different colours if the selection is without replacement.

- **Solution**

We could draw a tree diagram but it would be very large!
Instead we could think about the options:

Possibility	Probability
R then W then B	$\frac{3}{12}\times\frac{4}{11}\times\frac{5}{10}=\frac{1}{22}$
R then B then W	$\frac{3}{12}\times\frac{5}{11}\times\frac{4}{10}=\frac{1}{22}$
W then R then B	$\frac{4}{12}\times\frac{3}{11}\times\frac{5}{10}=\frac{1}{22}$
W then B then R	$\frac{1}{22}$
B then R then W	$\frac{1}{22}$
B then W then R	$\frac{1}{22}$

So P(different colours) = $\frac{6}{22}=\frac{3}{11}$