OCR Statistics 1
Solutions June 2005

1 (i)

Rank A	1	2	3	4	5
Rank B	4	1	3	2	5
d	-3	1	0	2	0
d^2	9	1	0	4	0

So $\sum d^2 = 14$

Therefore, $r_S = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 14}{5(25-1)} = 1 - 0.7 = 0.3$

(ii) If the rankings have a Spearman's rank correlation coefficient, then the rankings must be in reverse:

Rank B	4	1	3	2	5	
Rank C		2	5	3	4	1

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2 (i) T ~ Geo(0.14)

a) P(T = 5) = $0.86^4 \times 0.14 = 0.0766$ (3 SF)

b) P(T = 1) = 0.14
P(T = 2) = 0.86 × 0.14 = 0.1204
P(T = 3) = 0.86^2 × 0.14 = 0.1035
P(T = 4) = 0.86^3 × 0.14 = 0.0890
P(T = 5) = 0.0766 (from above)
P(T = 6) = 0.86^5 × 0.14 = 0.0659
P(T = 7) = 0.86^6 × 0.14 = 0.0566

Therefore P(T < 8) = 0.652

OR P(T < 8) = 1 – P(T > 7) = 1 – 0.86^7 = 1 – 0.348 = 0.652 (3 SF)

(ii) E(T) = 1/p = 7.14 (3 SF)

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3 (i) Let X = number of shoppers in the sample who buy washing powder.
Then X ~ B(16, 0.35).

a) P(X ≥ 8) = 1 – P(X ≤ 7) = 1 – 0.8406 = 0.1594

b) P(4 ≤ T ≤ 9) = P(T ≤ 9) – P(T ≤ 3) = 0.9771 – 0.1339 = 0.8432

(ii) Now, X ~ B(16, 0.38)
P(X = 6) = ${}^{16}C_6 \times 0.38^6 \times 0.62^{10} = 0.202$

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4 (i)

$$S_{xy} = 14464.10 - \frac{265 \times 274.6}{5} = -89.7$$

$$S_{xx} = 14176.54 - \frac{265^2}{5} = 131.54$$

$$S_{yy} = 15162.22 - \frac{274.6^2}{5} = 81.188$$

Therefore, $r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{-89.7}{\sqrt{131.54 \times 81.188}} = -0.868$ (3 sf)

(ii) The PMCC would be unchanged – the PMCC is unaffected by linear transformations of the variables.

(iii) We need to find the regression line of y on x.

$$b = \frac{S_{xy}}{S_{xx}} = \frac{-89.7}{131.54} = -0.682$$

Also, $\bar{y} = 54.92$, $\bar{x} = 53$.

So $a = \bar{y} - b\bar{x} = 54.92 - (-0.682) \times 53 = 91.1$

So the regression line is $y = 91.1 - 0.682x$

Substituting $x = 60.4$ gives $y = 91.1 - 0.682 \times 60.4 = 49.9$ cm

5 (i) UQ = 69
LQ = 45
So IQR = 24 marks

(ii) If 40% of candidates scored more than x marks, then 60% scored less than x.
60% of 1200 = 720.
Reading across at 720 gives $x = 63$.

(iii) Reading up from $x = 68$ gives a CF of 860.
So number of people scoring above 68 is 1200 – 860 = 340

(iv) The probability that a person scores more than 68 marks = 340 / 1200 = 0.2833
The probability that all 5 people score more than 68 marks = $0.2833^5 = 0.00183$

(v) The CF graph will now be a straight line between x = 35 and x = 55.
This means that the LQ will be smaller than that found in (i).
Therefore the IQR will be greater.

6 (i) $a = 4/5$, $b = 1/5$, $c = ¼$, $d = ¾$, $e = ¾$, $f = ¼$

(ii) $P(R = 2) = \left(\frac{1}{2} \times \frac{4}{5} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{5} \times \frac{1}{4}\right) + \left(\frac{1}{2} \times \frac{3}{5} \times \frac{3}{4}\right) = \frac{9}{20}$

(iii) $k = 1 - \frac{1}{10} - \frac{9}{20} - \frac{1}{5} = \frac{1}{4}$

(iv) $E(R) = \left(0 \times \frac{1}{10}\right) + \left(1 \times \frac{1}{4}\right) + \left(2 \times \frac{9}{20}\right) + \left(3 \times \frac{1}{5}\right) = 1.75$

E(R^2) = $\left(0^2 \times \frac{1}{10}\right) + \left(1^2 \times \frac{1}{4}\right) + \left(2^2 \times \frac{9}{20}\right) + \left(3^2 \times \frac{1}{5}\right) = 3.85$

Therefore, Var(R) = 3.85 – 1.75^2 = 0.7875

7 (i) Number of ways of choosing 7 people from 18 = ${}^{18}C_7 = 31824$

(ii) Number of ways of choosing 2 people from Gloucester = ${}^{5}C_2 = 10$

Number of ways of choosing 2 people from Hereford = ${}^{6}C_2 = 15$

Number of ways of choosing 3 people from Worcester = ${}^{7}C_3 = 35$

So total number of ways = $10 \times 15 \times 35 = 5250$

The probability is therefore 5250/31824 = 0.165

(iii) Number of ways of choosing 5 people from Worcester = ${}^{7}C_5 = 21$

Number of ways of choosing 2 people from Gloucester or Hereford = ${}^{11}C_2 = 55$

Total number of ways = 21 × 55 = 1155

So probability = 1155/31824 = 0.0363

(iv) There are 3 possibilities:

Case 1: 2 from G, 2 from H and 3 from W: ${}^{5}C_2 \times {}^{6}C_2 \times {}^{7}C_3$

Case 2: 2 from G, 3 from H and 2 from W: ${}^{5}C_2 \times {}^{6}C_3 \times {}^{7}C_2$

Case 3: 3 from G, 2 from H and 2 from W: ${}^{5}C_3 \times {}^{6}C_2 \times {}^{7}C_2$

These add to make 12600.

So probability is 12600/31824 = 0.396